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COMMENTS ON BLADE EXCITED RIGID BODY VIBRATIONS OF ROTARY VANE COMPRESSORS

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ABSTRACT

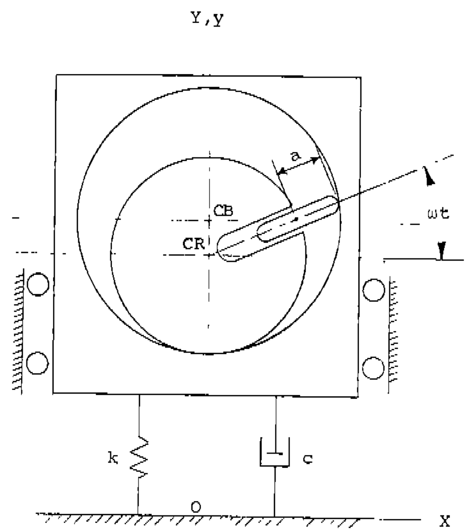
Without loss of **generality**, but illustrated on a single degree of freedom rigid body vibration model, the excitation action of the rotating blades of rotary vane compressors or motors is investigated for the case of a cylindrical bore. Significant findings are that four blade compressors and multiples of four are excitation free, while odd numbers of blades produce the highest harmonic content in the excitation function.

INTRODUCTION

From a vibration and noise control viewpoint, it is important to study the excitation mechanism of a rotary vane compressor or motor. While similar studies may have been carried out in the past, results of such studies in the open literature have never appeared, or, at least, could not be found by the authors. This is the justification for this technical note.

This paper describes an application of the classical problem of rotating unbalance to rotary sliding-vane compressors. Figure 1 shows a simplified model of a rotary compressor with a single vane.

Examining the figure, one can intuitively see the analogy between this problem and the classical rotating unbalance problem. The primary difference lies in the fact that in the situation pictured the length of the eccentricity arm varies with time. In the following paragraphs (1) the equation of motion of the model shown in Figure 1 will be derived from basic principles, (2) in a similar manner the equation of motion for the diametrically opposite two vane case will be derived from basic principles, (3) subsequently, it will be shown that the equation of motion for the double vane situation can be easily obtained from the single vane case and can be extended in general to an arbitrary number of vanes at various angles with respect to the initial blade, and finally (4) an interesting result for the four vane case and multiples of four will be presented.



xyz - Translating Reference Frame Attached to Rotor
XYZ - Fixed Reference Frame

Figure 1. Simplified Model of a Single-Vane, Rotary Sliding-Vane Compressor

EQUATION OF MOTION SINGLE-VANE CASE

Shown in Figure 2 are the primary geometric points of interest for the single vane case.

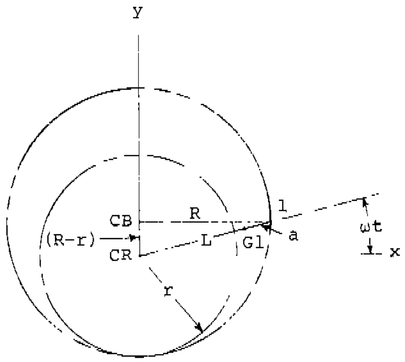
Our first task is to form the position vector of the center of mass (G1) of the vane with respect to the fixed coordinate system OXYZ.

$$\vec{r}_{G1/0} = \vec{r}_{CR/0} + \vec{r}_{G1/CR} \qquad (1)$$

The position vector, $\vec{r}_{G1/CR}$, is formed as follows:

$$\vec{r}_{G1/CR} = d\cos\omega t\ \vec{i} + d\sin\omega t\vec{j} \qquad (2)$$

where $d = L-a$.



CB - Center of Cylinder Bore
CR - Center of Rotor
1 - Contact Point of Sliding Vane with Cylinder Wall
R - Radius of Cylinder
r - Radius of Rotor
G1 - Center of Mass of Sliding Vane
L - Distance from CR to 1
a - Half Length of Sliding Vane

Figure 2. Geometric Points of Interest for Single-Vane Case

An expression for L which is valid for all time must be obtained. It is readily recognized that (CB,CR,1) form a triangle. Therefore, applying the law of cosines to this triangle we have

$$R^2 = (R-r)^2 + L^2 - 2L(R-r)\cos(90-\omega t)$$

Making the trigonometric substitution

$$\cos(90-\omega t) = \sin \omega t$$

and rearranging, we obtain

$$L^2 - [2(R-r)\sin\omega t]L + [r(r-2R)] = 0 \quad (3)$$

We solve for L by applying the quadratic equation and obtain

$$L = (R-r)\sin\omega t + \sqrt{[(R-r)\sin\omega t]^2 - r(r-2R)} \quad (4)$$

where the positive value of the radical was chosen since L must be positive. Therefore,

$$d = L - a = (R-r)\sin\omega t + \sqrt{[(R-r)\sin\omega t]^2 - r(r-2R)} - a \quad (5)$$

Substituting (5) into (2) and making a trigonometric substitution, we have

$$\vec{r}_{G1/CR} = \{\tfrac{1}{2}(R-r)\sin 2\ t + \sqrt{[(R-r)\sin\omega t]^2 - r(r-2R)}\ \cos\omega t - a\cos\omega t\}\vec{i} + \{(R-r)\sin^2\omega t + \sqrt{[(R-r)\sin\omega t]^2 - r(r-2R)}\ \sin\omega t - a\sin\omega t\}\vec{j}$$

Let

$$A(t) = \{\tfrac{1}{2}(R-r)\sin 2\omega t + \sqrt{[(R-r)\sin\omega t]^2 - r(r-2R)}\ \cos\omega t - a\cos\omega t\}$$

Therefore,

$$\vec{r}_{G1/CR} = A(t)\vec{i} + \{(R-r)\sin^2\omega t + \sqrt{[(R-r)\sin\omega t]^2 - r(r-2R)}\ \sin\omega t - a\sin\omega t\}\vec{j} \quad (6)$$

The position vector, $\vec{r}_{CR/0}$, is

$$\vec{r}_{CR/0} = [\Delta st + (Y+K)]\vec{j} \quad (7)$$

where

Δst - location of center of mass of housing and rotor combination at static equilibrium

Y - displacement of center mass of housing and rotor combination from static equilibrium

K - a constant distance from housing - rotor mass center to CR

Substituting (6) and (7) in (1) and combining similar unit vector terms, we have

$$\vec{r}_{G1/0} = Ai + \{Y + K^1 + (R-r)\sin^2\omega t + \sqrt{[(R-r)\sin\omega t]^2 - r(r-2R)}\ \sin\omega t - a\sin\omega t\}\vec{j} \quad (8)$$

where $K^1 = \Delta st + K = \text{constant}$.

Taking a dynamic free-body diagram of the entire system of housing, rotor, and blade in Figure 3,

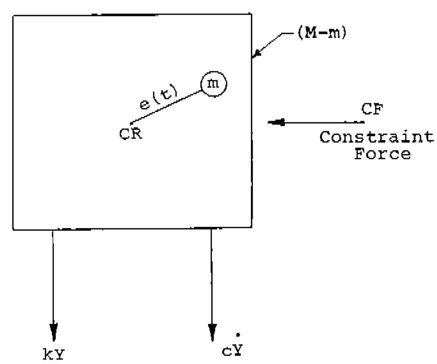


Figure 3. Dynamic Free-Body Diagram, Single-Vane Case

we apply Newton's 2nd Law for a system of particles, which is

$$\sum_{i=1}^{N} \vec{F}_{i_{external}} = \sum_{i=1}^{N} m_i \frac{d^2}{dt^2}(\vec{r}_{i/0}) \quad (9)$$

Applying this equation to the Y direction, we have

$$-kY-c\dot{Y}=(M-m)\ddot{Y}+m\frac{d^2}{dt^2}\{Y+K^1+(R-r)\sin^2\omega t+\sqrt{[(R-r)\sin\omega t]^2-r(r-2R)}\,\sin\omega t - a\sin\omega t\}$$

Rearranging and performing the differentiation for the first two terms, we obtain

$$M\ddot{Y}+C\dot{Y}+kY=m\frac{d^2}{dt^2}\{a\sin\omega t-(R-r)\sin^2\omega t-\sqrt{[(R-r)\sin\omega t]^2-r(r-2R)}\,\sin\omega t\} \tag{10}$$

Equation (10) is the governing differential equation of motion for the system. If we carry out the indicated differentiation, we obtain

$$\begin{aligned}M\ddot{Y}+C\dot{Y}+kY=m\Big\{&-2\omega^2(R-r)\cos 2\omega t-a\omega^2\sin\omega t\\&+\omega^2\sin\omega t\sqrt{[(R-r)\sin\omega t]^2-r(r-2R)}\\&-\frac{\omega^2(R-r)\sin 3\omega t}{\sqrt{[(R-r)\sin\omega t]^2-r(r-2R)}}\\&+\frac{(1/4)\omega^2(R-r)^2\sin\omega t\sin^2 2\omega t}{\{[(R-r)\sin\omega t]^2-r(r-2R)\}^{3/2}}\Big\}\end{aligned} \tag{11}$$

The forcing function could be expanded in series form and it could be shown that most harmonics of the fundamental excitation frequency corresponding to the rotational speed are present

EQUATION OF MOTION DOUBLE-VANE CASE

We now consider the diametrically opposite double-vane case shown in Figure 4.

Following the same procedure as before we obtain for d_1 and d_2

$$\begin{aligned}d_1&=(R-r)\sin\omega t+\sqrt{[(R-r)\sin\omega t]^2-r(r-2R)}-a\\d_2&=-(R-r)\sin\omega t+\sqrt{[(R-r)\sin\omega t]^2-r(r-2R)}-a\end{aligned} \tag{12}$$

The position vectors of the centers of mass of the two vanes with respect to the center of the rotor are

$$\begin{aligned}\vec{r}_{G1/CR}&=d_1\cos\omega t\vec{i}+d_1\sin\omega t\vec{j}\\\vec{r}_{G2/CR}&=-d_2\cos\omega t\vec{i}-d_2\sin\omega t\vec{j}\end{aligned} \tag{13}$$

Substituting (12) in (13) we obtain

$$\begin{aligned}\vec{r}_{G1/CR}=&\{(R-r)\sin\omega t\cos\omega t+\sqrt{[(R-r)\sin\omega t]^2-r(r-2R)}\cos\omega t-a\cos\omega t\}\vec{i}\\&+(R-r)\sin^2\omega t+\sqrt{[(R-r)\sin\omega t]^2-r(r-2R)}\sin\omega t-a\sin t\}\vec{j}\\\vec{r}_{G2/CR}=&\{(R-r)\sin\omega t\cos\omega t-\sqrt{[(R-r)\sin\omega t]^2-r(r-2R)}\cos\omega t+a\cos\omega t\}\vec{i}\\&+\{(R-r)\sin^2\omega t-\sqrt{[(R-r)\sin\omega t]^2-r(r-2R)}\sin t+a\sin\omega t\}\vec{j}\end{aligned} \tag{14}$$

At this point we attempt to find the position vector of the center of mass of the combined system of two vanes. Recalling

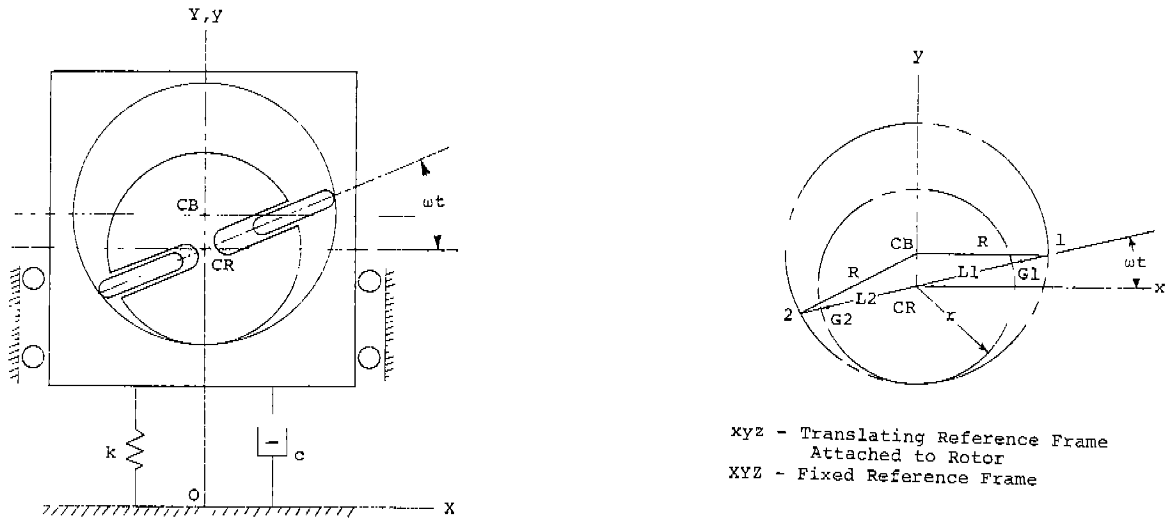


Figure 4. Simplified Model of a Double-Vane Rotary Sliding-Vane Compressor

from basic principles the definition of the center of mass of a system of particles

$$[\sum_{i=1}^{N} m_i]\vec{r}_{G/0} = \sum_{i=1}^{N} m_i \vec{r}_{i/0} \tag{15}$$

For this particular case with $m_1=m_2=m$, we have

$$2m\vec{r}_{G/CR} = m\vec{r}_{G1/CR}+m\vec{r}_{G2/CR}$$

$$\vec{r}_{G/CR} = \frac{\vec{r}_{G1/CR}+\vec{r}_{G2/CR}}{2} \tag{16}$$

Substituting (14) in (16) results in

$$\vec{r}_{G/CR}=(R-r)\sin\omega t\cos\omega t\vec{i}+(R-r)\sin^2\omega t\vec{j} \tag{17}$$

Therefore, the position vector of the center of mass of the system of two vanes with respect to the fixed reference 0XYZ is

$$\vec{r}_{G/0} = \vec{r}_{CR/0} + \vec{r}_{G/CR}$$

$$\vec{r}_{G/0}=\frac{1}{2}(R-r)\sin 2\omega t\vec{i}+[Y+K^1+(R-r)\sin^2\omega t]\vec{j} \tag{18}$$

Applying Newton's 2nd Law to the system of housing rotor and vanes shown in Figure 5

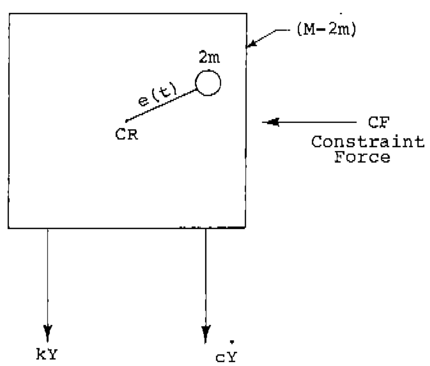


Figure 5. Dynamic Free-Body Diagram, Double-Vane Case

we have for the Y direction

$$M\ddot{Y} + C\dot{Y} + kY = -2m\frac{d^2}{dt^2}[(R-r)\sin^2\omega t] \tag{19}$$

Performing the indicated differentiation we have for the governing differential equation of motion

$$M\ddot{Y} + C\dot{Y} + kY = -4m\omega^2(R-r)\cos 2\omega t \tag{20}$$

The well known, but still interesting result is that in the two vane case only the second harmonic of the rotation frequency excites the compressor.

DERIVATION OF DOUBLE-VANE EQUATION OF MOTION FROM SINGLE-VANE CASE

Let us now obtain the governing Equation (19) for the double-vane case from Equation (10), the governing equation for the single vane case. To be more specific, we will attempt to obtain Equation (19) by adding to the right hand side of (10) a second mass with angular position increased by π. We obtain

$$\begin{aligned} M\ddot{Y}+C\dot{Y}+kY = {} & m\frac{d^2}{dt^2}\{a\sin\omega t-(R-r)\sin^2\omega t \\ & - \sqrt{[(R-r)\sin\omega t]^2-r(r-2R]}\ \sin\omega t\} \\ & + m\frac{d^2}{dt^2}\{a\sin(\omega t+\pi)-(R-r)\sin^2(\omega t+\pi) \\ & - \sqrt{[(R-r)\sin(\omega t+\pi]^2-r(r-2R)}\ \sin(\omega t+\pi)\} \end{aligned} \tag{21a}$$

$$\begin{aligned} M\ddot{Y}+C\dot{Y}+kY = {} & m\frac{d^2}{dt^2}\{a\sin\omega t+a\sin(\omega t+\pi) \\ & - (R-r)\sin^2\omega t - (R-r)\sin^2(\omega t+\pi) \\ & - \sqrt{[(R-r)\sin\omega t]^2-r(r-2R)}\ \sin\omega t \\ & - \sqrt{[(R-r)\sin(\omega t+\pi)]^2-r(r-2R)}\ \sin(\omega t+\pi)\} \end{aligned} \tag{21b}$$

Making the trigonometric substitution

$$\sin(\omega t+\pi) = -\sin\omega t$$

(21b) reduces to

$$M\ddot{Y} + C\dot{Y} + kY = -2m\frac{d^2}{dt^2}[(R-r)\sin^2\omega t] \tag{19}$$

This is the same equation as obtained by application of basic principles. Through the same process (20) can be obtained from (11). If this process is continued for higher number of vanes, one would find that the process described is totally general for extension to an arbitrary number of vanes at various angles with respect to the initial vane.

EQUATION OF MOTION FOUR-VANE CASE

A very interesting situation occurs when we consider the four vane compressor shown in Figure 6.

Applying the superposition technique described previously we obtain

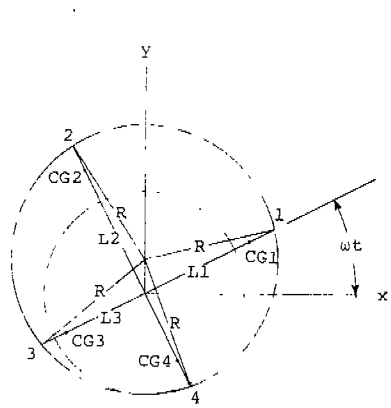


Figure 6. Simplified Model Four-Vane Rotary Sliding-Vane Compressor

$$M\ddot{Y}+C\dot{Y}+kY = m\frac{d^2}{dt^2}\{a\sin\omega t-(R-r)\sin^2\omega t$$

$$-\sqrt{[(R-r)\sin\omega t]^2-r(r-2R)}\ \sin\omega t\}$$

$$+m\frac{d^2}{dt^2}\{a\sin(\omega t+\tfrac{\pi}{2}) - (R-r)\sin^2(\omega t+\tfrac{\pi}{2})$$

$$-\sqrt{[(R-r)\sin(\omega t+\tfrac{\pi}{2})]^2-r(r-2R)}\ \sin(\omega t+\tfrac{\pi}{2})\}$$

$$+m\frac{d^2}{dt^2}\{a\sin(\omega t+\pi)-(R-r)\sin^2(\omega t+\pi)$$

$$-\sqrt{[(R-r)\sin(\omega t+\pi)]-r(r-2R)}\ \sin(\omega t+\pi)\}$$

$$+m\frac{d^2}{dt^2}\{a\sin(\omega t+\tfrac{3\pi}{2}) - (R-r)\sin^2(\omega t+\tfrac{3\pi}{2})$$

$$-\sqrt{[(R-r)\sin(\omega t+\tfrac{3\pi}{2})]-r(r-R)}\ \sin(\omega t+\tfrac{3\pi}{2})\} \quad (22)$$

Using the following trigonometric substitution

$$\sin(\omega t+\tfrac{\pi}{2}) = \cos\omega t$$

$$\sin(\omega t+\pi) = -\sin\omega t$$

$$\sin(\omega t+\tfrac{3\pi}{2}) = -\cos\omega t$$

Equation (22) reduces to

$$M\ddot{Y}+C\dot{Y}+kY = m\frac{d^2}{dt^2}[-2(R-r)]=-4m\frac{d^2}{dt^2}[\tfrac{1}{2}(R-r)]$$

or

$$M\ddot{Y} + C\dot{Y} + kY = 0 \quad (23)$$

The preceding equation indicates that for the four-vane case no rotating unbalance due to the vanes exists. To understand why this situation occurs one must return to basic principles and form the position vector of the center of mass of the system of four vanes with respect to the center of the rotor. If this is done, the resulting position vector is found to be

$$\vec{r}_{G/CR} = \frac{1}{2}(R-r)\vec{j} \quad (24)$$

This implies that for the four vane case the location of the center of mass of the system of four vanes is midway between the center of the rotor (CR) and the geometric center of the cylinder bore (CB) for all time. This is shown in Figure 7.

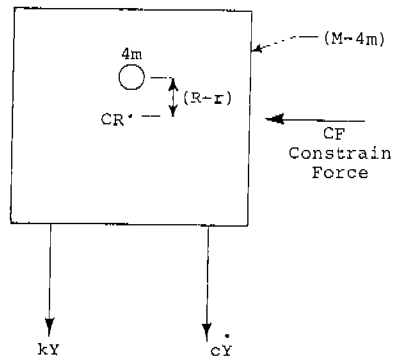


Figure 7. Dynamic Free-Body Diagram Four-Vane Case

From a physical standpoint the mathematics tell us that due to the geometry of the system, the net contributions of all blades considered together results in a cancellation of the unbalancing effect. If one were to examine the eight-vane case, one would see that this situation is repeated.

DISCUSSION AND CONCLUSION

When discussing the excitation of a rotary vane compressor or motor by its blades, it should be remembered that this is often only a secondary effect, especially as the number of vanes increases. The primary effect may very well be an unbalance of the compressor and electromotor rotor, which of course always occurs at the frequency corresponding to the rotational speed. However, the primary unbalance of a rotary vane compressor can be completely eliminated, in contrast to the reciprocating compressor, where this is economically impossible since it would require a gearing system. In this case, all that is left is the blade unbalance, and one is justified to take a closer look at it.

A second point is that this paper confined itself to equally spaced blades of equal mass. In the real world manufacturing tolerances interfere. Thus, one would suspect that a careful Fourier analysis of experimental data taken from, for instance, a four bladed compressor would still reveal small response spikes at frequencies where there should be none, however, this does not invalidate the general trend predicted by the theory.

In conclusion, the paper has presented the vibration excitation function for rotary vane compressors of any number of blades, but has discussed mainly the single, double and quadruple blade cases. The approach is valid for equally spaced blades, but, of course, also for unequal spacing, since the excitation function is generated by a superposition of single blade cases. While it might be of interest to repeat this analysis for the case where the compressor is allowed to move in three or even six degrees of freedom, the single degree of freedom case is sufficient to illustrate how the number and spacing of blades affects the vibration excitation.